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Executive Summary

SPEEDD (Scalable ProactivE Event-Driven Decision Making) is developing a system for event recognition and proactive decision making in real-time, based on the on-the-fly processing of Big-Data streams. The goal of WP4 (Real-Time Event-Based Decision-Making Under Uncertainty) is to provide innovative techniques for proactive event-driven decision-making, ranging from worst-case real-time decision-making to randomized, scenario-based decision-making, enabling varying degrees of automation.

The purpose of this document is to describe the first version of the decision making component. This component provides a body of proactive event-driven decision-making tools for the respective use-cases, which exploit the detected or forecasted events of complex event processing. Conceptually, the Decision Making module receives as inputs the detected, derived and forecasted events and emits control actions or appropriate suggestions. Note that any decision making algorithms that are expected to satisfy guarantees on optimality and robustness can only provide those guarantees with respect to an underlying model. Hence those decision making tools need to be inherently use-case specific. In this deliverable, we describe decision making tools for freeway traffic control, inner-city traffic control and credit card fraud detection.

The main contributions of this deliverable address the traffic use case. A theoretical analysis of an existing decentralized decision making algorithm for freeways (ALINEA) is presented and optimality guarantees are derived. This decision making scheme has been implemented within the event-driven, scalable architecture selected for the SPEEDD project. The implemented algorithms will also be part of the traffic show-case for the first review meeting. Furthermore, a novel algorithm for the optimization of large-scale urban traffic networks has been derived. Performance measures for the use-case objectives are introduced and simulation results are presented, which show clear improvements. To address the second use case (Credit-Card Fraud Detection), we introduce the novel concept of "distributionally robust classifiers". This approach mitigates the uncertainty in the training data in a "worst-case" approach.

Future extensions for the decision making strategies for the traffic use case will introduce coordination and online optimization into the decision making process. They are expected to build upon the deentralized algorithm in a hierarchical fashion. Distributionally robust classifiers in their present stage are linear classifiers. To improve performance, it is worth exploring if they can be generalized to allow for nonlinear decision boundaries by employing the kernel trick or an equivalent extension.

1

Introduction

1.1 History of the Document

Version	Date	Author	Change Description
0.1	01/12/2014	Marius Schmitt (ETH)	Initial draft
0.2	01/12/2014	Marius Schmitt (ETH)	Included contribution by P. Grandinetti (CNRS)
0.3	19/12/2014	Marius Schmitt (ETH)	Refined draft for internal review
1.0	23/12/2014	Marius Schmitt (ETH)	Addressed some comments by the reviewers

1.2 Purpose and Scope of the Document

The purpose of this document is to describe the first version of the decision making component. We describe decision making tools for freeway traffic control, inner-city traffic control and credit card fraud detection. The deliverable is structured as follows:

In Chapter 2, a novel, theoretical analysis of an existing, decentralized decision making algorithm for freeways (ALINEA) is presented. The central result is that this (simplistic) feedback controller is asymptotically flow-optimal for the idealized freeway ramp-metering problem. Because of its inherently distributed structure, it has been selected for a first implementation within the SPEEDD project. It is intended to serve as a bottom-level controller for the freeway use case, on top of which coordinated, optimization-based decision making strategies can be implemented. Note that even though decisions are made without the explicit use of robust optimization tools (T4.2: "Worst-Case Decision-Making Methods"), the inherent feedback structure of the control strategies mitigates the uncertainties in freeway traffic to a certain extent. In particular, ALINEA only requires one model parameter (the critical density) to be estimated a priori for implementation. Domain-specific triggering mechanisms based on the reported events as outlined in T4.1 ("Event-Driven Proactive Decision-Making") are in place.

Chapter 3 describes the design of a novel algorithm for optimization of large-scale urban traffic networks. The algorithm has favorable scalability properties, because it formulates the decision making problem as a linear program. Performance measures for the use–case objectives are introduced and an initial evaluation of simulation results is given.

In the second use-case (Credit Card Fraud Detection), Event Recognition (WP3) and Decision Making (WP4) are much closer related, since the decision of whether a transaction should be blocked obviously depends on the estimated probability that the respective transaction is fraudulent. Therefore, we briefly address the classification problem in this deliverable as well and we present a novel linear classifier in Chapter 4. We introduce the concept of “distributionally robust classifiers”, which are distribution-agnostic as opposed to existing solutions that commonly rely on the assumption of Gaussian probability distributions. The uncertainty, which is introduced into the problem by the absence of an exact knowledge of the probability density function, is then handled using a “worst-case” (T4.2) approach.

1.3 Relationship with Other Documents

This document takes the scenario definitions given in D8.1 for the Proactive Traffic Management use case and D7.1 for the Proactive Credit Card Fraud Detection use case into account. We also refer to the description of the architecture design of the SPEEDD prototype (D6.1)in section 2.4, where we briefly address the integration of the decision making module into the SPEEDD framework. The development of decision making components is ongoing work, and future deliverables (D4.2 and D4.3) will build upon the initial results presented in this document. Parts of the results presented in this document have been submitted for publication in Schmitt et al.. and Grandinetti et al..

2

Flow-maximizing Equilibria of the CTM

M. Schmitt, P. Goulart, J. Lygeros

We consider the freeway ramp metering problem, based on the Cell Transmission Model. This work addresses the question of how well distributed control strategies, e.g. local feedback controllers at every onramp, can maximize the traffic flow asymptotically under time-invariant boundary conditions. We extend previous results on the structure of steady-state solutions of the Cell Transmission Model and use them to optimize over the set of equilibria. By using duality arguments, we derive optimality conditions and show that closed-loop equilibria of certain distributed feedback controllers, in particular the practically successful "ALINEA" method, are in fact globally optimal. Parts of this chapter have been submitted for publications in Schmitt et al..

2.1 Introduction

Active traffic control schemes have been established as an effective and practically useful tool to improve traffic flows on congestion-prone road networks Papageorgiou et al. (2003). This work concentrates on the freeway ramp metering problem, where we can actively control the number of cars that enter the freeway using a specific onramp. A survey of ramp metering strategies can be found in Papageorgiou and Kotsialos (2000). To model the freeway traffic dynamics, we make use of the Cell Transmission Model (CTM), which was originally derived as a first-order Godunov approximation of the kinematic wave partial differential equation Daganzo (1994, 1995). More precisely, we adopt the "asymmetric" CTM Gomes and Horowitz (2006); Gomes et al. (2008), which simplifies the model of onramp-mainline merges in comparison to the originally proposed formulation. Its popularity for model-based control stems from the simplicity of the model equations, allowing for computationally efficient solutions methods for optimal control problems Gomes and Horowitz (2006); Ziliaskopoulos (2000).

A variety of local feedback strategies, i.e., ramp metering controllers that only receive measurements from sensors in close vicinity of the onramp location, have been described in literature, e.g. Papageorgiou et al. (1991); Stephanedes (1994); Zhang and Ritchie (1997). These strategies have been shown to come close to the performance of optimal control strategies in practical applications, even though they

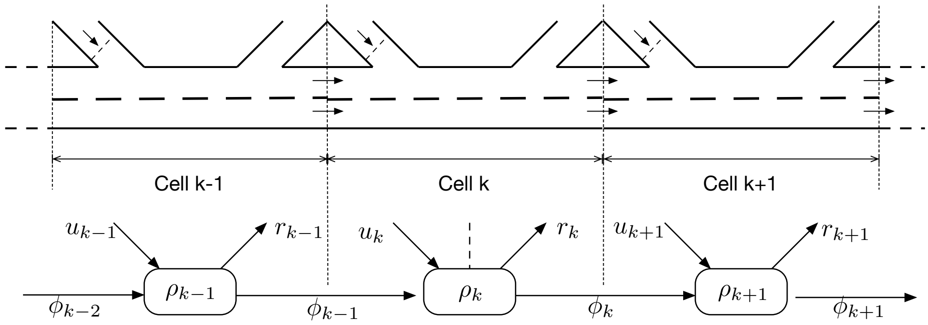


Figure 2.1: Sketch of the cell transmission model

only aim to maximize bottleneck flows locally Smaragdis et al. (2004); Wang et al. (2014). While it is apparent that such local feedback controllers are far easier to implement and to configure than centralised, model-based optimal control strategies, it is not obvious why the former tend to come close in performance to the latter in practice Papamichail et al. (2010). An explanation is given in Zhang and Levinson (2004), which explicitly constructs the optimal control strategy for a special case – it is assumed for example that there are no internal freeway queues – and states that the structure of the explicit solution "explains why some local metering algorithms [...] are successful – they are really close to the most-efficient logic."

In this work, we address the question of how distributed control strategies, such as local feedback controllers at every onramp, compare to optimal control strategies asymptotically under time-invariant boundary conditions. The idea to focus on traffic controllers which are only required to achieve convergence to an optimal equilibrium (instead of solving the far more challenging problem of optimizing the transient behavior) has recently received attention in a series of papers on traffic density balancing Pisarski and C. de Wit (2012, 2013); Pisarski and De Wit (2012). It is well known that the problem of maximizing the flow over the set of equilibria of the CTM can be posed as a linear program (LP). By using duality arguments, we derive simple optimality conditions for flow-maximizing equilibria. We show that all closed-loop equilibria of certain local feedback controllers, in particular the practically successful ALINEA method Papageorgiou et al. (1991), are in fact globally optimal in the idealized CTM.

2.2 Problem formulation

In this section, we motivate and introduce the (asymmetric) Cell Transmission Model (CTM) Gomes and Horowitz (2006); Gomes et al. (2008). Throughout this work, we consider a freeway section as depicted in Figure 2.1. The CTM admits the following intuitive explanation: The freeway is partitioned into n sections or *cells* of length l_k. The state of the highway is described by the traffic *density* $\rho_k(t)$ in each cell k at sampling time t. Since the CTM is a first order model, the velocity is not part of the state. The evolution of the traffic is described by the traffic *flows* $\phi_k(t)$, i.e., the number of cars that move from cell k to cell $k+1$ in one time interval Δt. We model the *off-ramp flows* $r_k(t) = \frac{\beta_k}{\bar{\beta}_k}\phi_k(t)$ as a proportion of the mainline flow $\phi_k(t)$ with the (constant) *split ratios* β_k and $\bar{\beta}_k := 1 - \beta_k$. The flow entering the freeway via an onramp at cell k is denoted u_k. The flow entering the considered freeway

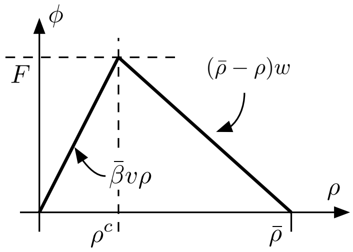


Figure 2.2: Sketch of the fundamental diagram for uniform freeway conditions, i.e., parameters and densities that do not differ between consecutive cells.

section on the mainline is denoted u_0. We can thus formulate the conservation law for each cell as:

$$\rho_k(t+1) = \rho_k(t) + \frac{\Delta t}{l_k}\left(\phi_{k-1}(t) + u_k(t) - \frac{\phi_k(t)}{\bar{\beta}_k}\right).$$

The road conditions are described by the so-called *free-flow velocity* v_k, the *congestion-wave velocity* w_k and the *jam density* $\bar{\rho}_k$. The flows $\phi_k(t)$ are limited by the number of cars in the origin cell that want to travel downstream $(\bar{\beta}_k v_k \rho_k(t))$, the *capacity* of the highway F_k and the available free space $((\bar{\rho}_{k+1} - \rho_{k+1}(t))w_{k+1})$ in the receiving cell:

$$\phi_k(t) = \min\left\{\bar{\beta}_k v_k \rho_k(t), F_k, (\bar{\rho}_{k+1} - \rho_{k+1}(t))w_{k+1}\right\}.$$

This relationship can be visualized with the so-called fundamental diagram as depicted in Figure 2.2. The *critical density* $\rho_k^c = \frac{w_k}{v_k+w_k}\bar{\rho}_k$ is the density value at which the fundamental diagram is maximized. The peak value of both the sending as well as the receiving cell determine the values of the capacities F_k, which are defined as $F_k := \min\left\{\bar{\beta}_k v_k \rho_k^c, (\bar{\rho}_{k+1} - \rho_{k+1}^c)w_{k+1}\right\}$ [1] . This equation is slightly adapted for the first and the last cell, where we define $F_0 = (\bar{\rho}_1 - \rho_1^c)w_1$ and $F_n = \min\left\{\bar{\beta}_n v_n \rho_n^c, \bar{\phi}_n\right\}$. Here, $u_0 \geq 0$ models the *mainline traffic demand* and $\bar{\phi}_n \geq 0$ is some arbitrary, constant bound on the outflow from the highway. All parameters and variables of the CTM are summarized in Table 2.1 . Note that all parameters of the CTM are positive and all states are nonnegative. Furthermore, the split ratios are limited to the interval $0 \leq \beta_k < 1$ and the sampling time Δ_t is restricted to $\Delta_t \leq \frac{l_k}{v_k}$ to ensure convergence.

For a given initial state $\rho_k(0)$ and ramp metering rates $u_k(t)$ for $1 \leq k \leq n$ and $t \in \mathbb{N}$, the evolution of the highway is described by the following equations:

$$\begin{aligned}
\rho_k(t+1) &= \rho_k(t) + \frac{\Delta t}{l_k}\left(\phi_{k-1}(t) + u_k(t) - \frac{\phi_k(t)}{\bar{\beta}_k}\right), \\
\phi_0(t) &= \min\left\{u_0, F_0, (\bar{\rho}_1 - \rho_1(t))w_1\right\}, \\
\phi_k(t) &= \min\left\{\bar{\beta}_k v_k \rho_k(t), F_k, (\bar{\rho}_{k+1} - \rho_{k+1}(t))w_{k+1}\right\}, \\
\phi_n(t) &= \min\left\{\bar{\beta}_n v_n \rho_n(t), F_n, \bar{\phi}_n\right\}.
\end{aligned}$$

Note that this model includes the implicit assumption that congestion does not spill back onto the on-ramps. While this assumption might not be satisfied for an uncontrolled highway, it was shown to be satisfied by a large margin for a highway controlled by ramp metering in a field study Gomes and

[1] Note that some authors allow for capacities $0 < F_k \leq \min\left\{\bar{\beta}_k v_k \rho_k^c, (\bar{\rho}_{k+1} - \rho_{k+1}^c)w_{k+1}\right\}$ which leads to a fundamental diagram of trapezoidal shape.

	Symbol	Name/Definition	Unit
Variables	ϕ_k	flow	1/h
	ρ_k	density	1/mile
	u_k	onramp flow	1/h
Parameters	β_k	split ratio	1
	$\bar{\beta}_k$	$1 - \beta_k$	1
	l_k	cell length	mile
	v_k	free-flow velocity	mile/h
	w_k	congestion-wave velocity	mile/h
	$\bar{\rho}_k$	jam density	1/mile
	Δ_t	sampling time interval	h

Table 2.1: Summary of symbols

Horowitz (2006), since such a controller will limit inflows and mainline congestion by design.

In this work, we seek to find an equilibrium of the CTM, which maximizes a positive, linear combination of the mainline flows:

$$\begin{aligned} &\underset{u_k,\phi_k,\rho_k}{\text{maximize}} && \textstyle\sum_{k=0}^{n} c_k \phi_k \\ &\text{subject to} && \phi_k, \rho_k, u_k \text{ describe an equilibrium of the CTM,} \end{aligned} \tag{2.1}$$

for $c_k \geq 0$. This somewhat informal problem statement will be made precise in Section 2.2.3, after we have analyzed the equilibria of the CTM in Section 2.2.1. We restrict our attention to equilibria, since periodic solutions do not provide an advantage with respect to flow–maximization (Section 2.2.2).

2.2.1 Existence of Equilibria

In this work, we are mainly interested in optimal steady-state solutions to the CTM equations. We start by deriving some general properties in the form of three lemmas that characterize equilibria of the CTM, which will be important tools in the analysis of optimality of such equilibria in the following sections. In the following, if the time index is omitted for some variable, e.g. ϕ_k instead of $\phi_k(t)$, the variable is to be understood as a steady-state value.

The equations describing steady-state solutions or equilibria of the CTM can be derived by imposing $\rho_k(t+1) = \rho_k(t) =: \rho_k$ (and removing the time index t from all variables), which yields:

$$\begin{aligned} \phi_k &= (\phi_{k-1} + u_k)\,\bar{\beta}_k, && 1 \leq k \leq n, \\ \phi_0 &= \min\{u_0, F_0, (\bar{\rho}_1 - \rho_1)w_1\}, \\ \phi_k &= \min\left(\bar{\beta}_k v_k \rho_k, F_k, (\bar{\rho}_{k+1} - \rho_{k+1})w_{k+1}\right), && 1 \leq k < n, \\ \phi_n &= \min\left\{\bar{\beta}_n v_n \rho_n, F_n, \bar{\phi}\right\}. \end{aligned} \tag{2.2}$$

We call u the (traffic) *demand*, which consists of the mainline demand u_0 and the onramp inflows u_k, $1 \leq k \leq n$ (as introduced earlier). Note that the onramp flows can be changed by ramp metering and thus serve as control inputs, whereas the mainline demand is fixed. For ease of notation, we define the *equilibrium set* $\mathcal{E}(u)$ as

$$\mathcal{E}(u) = \{(\phi, \rho) : \text{For fixed demand } u, (\phi, \rho) \text{ satisfy (2.2)}\}.$$

We call a section k a *bottleneck* if $\phi_k = F_k$. The locations of the bottlenecks are important in the analysis of the CTM equilibria Gomes et al. (2008). Assuming that there are $m-1$ bottlenecks $b_1, b_2, \ldots, b_{m-1}$, these bottlenecks partition the highway into m *segments* $S_1 = \{0, \ldots, b_1\}$, $S_2 = \{b_1+1, \ldots, b_2\}, \ldots, S_m = \{b_{m-1}+1, \ldots, n\}$, as depicted in Figure 2.3. Note that the first and the last segment may be empty if ϕ_0 or ϕ_n are bottleneck flows. For a constant demand u, define the *induced*

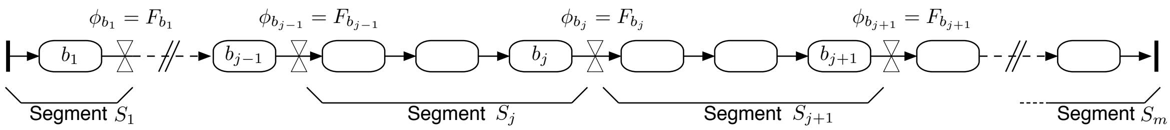


Figure 2.3: Segments, defined according to the bottleneck locations, in a CTM representation of a freeway.

flows φ as:

$$\varphi := u_0,$$
$$\varphi_k := (\varphi_{k-1} + u_k)\bar{\beta}_k, \qquad 1 \leq k \leq n,$$

i.e., the induced flows are the flows that result in steady-state if the complete traffic demand can be accommodated by the freeway.

We can categorize traffic demands for a particular freeway according to whether or not they can be served: The traffic demand u is called *feasible* if the induced flows are equal or smaller than the local capacities in every section $\varphi_k \leq F_k \ \forall k$. It is called *strictly feasible* if the induced flows are strictly smaller than the capacity in every section $\varphi_k < F_k \ \forall k$ and it is called *marginally feasible* if it is feasible, but not strictly feasible. Using these definitions, we are now ready to characterize equilibria of the CTM.

Lemma 1. *For a feasible traffic demand u, the unique equilibrium flows of the CTM are equal to the induced flows $\phi_k = \varphi_k$. Equilibrium densities are as follows:*

(i) For strictly feasible demands, the unique equilibrium is the uncongested equilibrium with densities $\rho_k = \frac{\phi_k}{\beta_k v_k} < \rho_k^c$.

(ii) For marginally feasible demands, the equilibrium densities are no longer unique. One particular equilibrium is the uncongested equilibrium with densities $\rho_k = \frac{\phi_k}{\beta_k v_k} \leq \rho_k^c$.

Proof. This result follows immediately from (Gomes et al., 2008, Theorem 4.1). □

Note that (Gomes et al., 2008, Theorem 4.1) also states the complete set of equilibrium densities for case (ii) explicitly. For our purposes, it is sufficient to know that a non-empty set of equilibrium densities exists for every feasible demand. We also need to consider situations in which the mainline demand cannot be completely served and a congestion on the mainline spills back outside of the considered part of the freeway. If the demand $u = \{u_0, u_1, \ldots, u_n\}$ is infeasible, but the demand $\tilde{u} = \{0, u_1, \ldots, u_n\}$ is feasible, we call the demand u *onramp-feasible*. Onramp-feasible demands exceed the capacity of the freeway, but the capacity is sufficient to accommodate the onramp flows alone, assuming zero mainline-flow.

Lemma 2. *For onramp-feasible demands, the unique equilibrium flow is given as $\varphi_0 = \max\{x \geq 0 : (x, u_1, \ldots, u_n)$ is feasible$\}$) and $\varphi_k := (\varphi_{k-1} + u_k)\bar{\beta}_k$ for $1 \leq k \leq n$. The equilibrium densities are not unique in general. One particular equilibrium is given as a congested first segment $\rho_k = \bar{\rho}_k - \frac{\phi_k}{w_k} \geq \rho_k^c$, $\forall k \in S_1$ and the remaining freeway operating in free-flow $\rho_k = \frac{\phi_k}{\beta_k v_k} \leq \rho_k^c$, $\forall k \notin S_1$.*

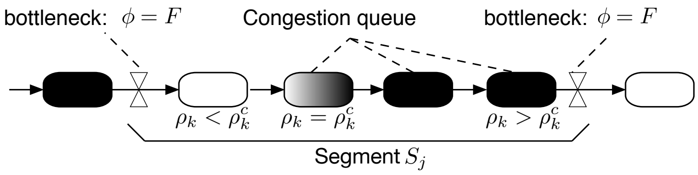


Figure 2.4: Pattern of the highway state within one segment. Note that there is not necessarily a cell in which the critical density is exactly achieved ($\rho_k = \rho_k^c$). In this case, cell q_i is the last cell in free flow: $\rho_k < \rho_k^c$.

Proof. See Schmitt et al.. □

In an equilibrium, some cells in a segment might be congested while other cells operate in free-flow. The order of congestion and free-flow conditions is not arbitrary, but follows a specific pattern instead. Let cell q_j be the last cell within segment S_j which is not congested, i.e. $q_j := \text{argmax}_k\{k \in S_j \,:\, \rho_k \leq \rho_k^c\}$. If all cells within a segment j are congested, define $q_j := b_j + 1$.

Lemma 3. *All cells downstream of cell q_j within a segment are congested: $\rho_k > \rho_k^c,\ k \in S_j, k > q_j$. All cells upstream of cell q_j within a segment operate in free-flow: $\rho_k < \rho_k^c,\ k \in S_j, k < q_j$.*

Proof. The first assertion holds by definition of q_j. The second assertion can be proven by contradiction: Assume some cell(s) upstream of cell q_j, but within the same segment are congested. Let $n_j := \text{argmax}_k\{k \in S_j, k < q_j : \rho_k \geq \rho_k^c\}$ be the most downstream one of those cells. By definition, cell n_j is congested, but cell n_j+1 is not. It follows that $\phi_{n_j} = \min\left\{\bar{\beta}_{n_j} v_{n_j} \rho_{n_j}, F_{n_j}, (\bar{\rho}_{n_j+1} - \rho_{n_j+1}) w_{n_j+1}\right\} \geq \min\left\{\bar{\beta}_{n_j} v_{n_j} \rho^c_{n_j}, F_{n_j}, (\bar{\rho}_{n_j} - \rho^c_{n_j}) w_{n_j}\right\} = F_{n_j}$, i.e. cell $n_j + 1$ is a bottleneck. This is a contradiction to the assumption that there are no bottlenecks within a segment. □

Intuitively, this means that there exists only one congestion queue per segment[2], which starts in cell q_j. Furthermore, there can at most be one cell within a segment (cell q_j) in which the density actually equals the critical density. The resulting congestion/ free-flow pattern is visualized in Figure 2.4.

Remark 1. *Note that the equilibrium flows as a function of the traffic demand $\phi(u)$ can be written as*

$$
\begin{aligned}
&\phi_0 = \max\{\varphi : \varphi \leq u_0,\ (\varphi, u_1, \ldots, u_n)\}, \\
&\phi_k = (\phi_{k-1} + u_k)\bar{\beta}_k, && 1 \leq k \leq n,
\end{aligned}
$$

for both feasible demands, yielding simply $\phi_0 = u_0$, and onramp-feasible demands, by definition.

Having established various properties of the equilibrium flows and densities for fixed onramp inflows u, we are now ready to address the problem of finding the optimal ramp metering rates u to maximize traffic flows.

2.2.2 Periodic solutions

In Problem (2.4), we optimize over the set of equilibria of the CTM equations in order to obtain the optimal steady-state solution. In Gomes et al. (2008) the authors speculate about the possibility, that the

[2] A similar result is stated in (Gomes et al., 2008, Lemma 4.3). The cases $\rho_k < \rho_k^c$ and $\rho_k = \rho_k^c$ are not distinguished, however, but this distinction will be crucial in our further analysis.

periodic execution of a congestion-decongestion cycle could potentially yield an improvement over the optimal equilibrium and thus provide what they call a ”free-lunch” opportunity. In the following, we will show that such a periodic cycle will **not** improve the steady-state optimum given as the solution to Problem (2.4), assuming that the boundary conditions $\underline{u}_k, \bar{u}_k, \phi_{in}$ and ϕ_{out} are constant in time.

To make this statement precise, let $u_k(t)$ be the inputs/inflows during one cycle/horizon of length N, i.e. $[u_k(0), u_k(1), \dots u_k(N-1)]^\top$. Let $\rho_k(t)$ be the corresponding densities, with $\rho_k(0)$ the initial condition, and $\phi_k(i)$ the corresponding flows. We can write down the CTM equations for one cycle:

$$\rho_k(t+1) = \rho_k(i) + \frac{\Delta t}{l_k}\left(\phi_{k-1}(t) + u_k(t) - \frac{\phi_k(t)}{\bar{\beta}_k}\right) \quad (2.3a)$$

$$\phi_k(t) = \min\left\{\bar{\beta}_k v_k \rho_k(t), F_k, (\bar{\rho}_{k+1} - \rho_{k+1}(t))w_{k+1}\right\} \quad (2.3b)$$

$$\phi_0(t) = \min\left\{F_0, (\bar{\rho}_1 - \rho_1(t))w_1\right\} \quad (2.3c)$$

$$\phi_n(t) = \min\left\{\bar{\beta}_n v_n \rho_n(t), F_{n+1}\right\} \quad (2.3d)$$

Since we are considering a periodic cycle, we impose in addition that $\rho_k(0) = \rho_k(N)$. Let us define the average inputs, flows and densities as

$$u_k^{avg} := \frac{1}{N}\sum_{i=0}^{N-1} u_k(t)\ ,$$

$$\phi_k^{avg} := \frac{1}{N}\sum_{i=0}^{N-1} \phi_k(t)\ ,$$

$$\rho_k^{avg} := \frac{1}{N}\sum_{i=0}^{N-1} \rho_k(t)$$

Theorem 1. *For any cycle of (finite) period N which operates in quasi-steady state, i.e. $\rho_k(0) = \rho_k(N)$ and satisfies the CTM equations, there exists a steady state solution to the CTM, that achieves both equal average onramp inflows in all sections $u_k^{avg} = u_k$ and also equal flows on the mainline $\phi_k^{avg} = \phi_k$, for all k. Moreover, any box constraints on the metering rates that are satisfied throughout the cycle will also be satisfied in the equilibrium. In this sense, there is no free lunch.*

Proof. Let $\phi_k(t)$ for $k \in \{0, \dots, N-1\}$ denote the flows during one periodic cycle of period N and accordingly $\rho_k(t)\ \forall k \in \{0, \dots, N-1\}$ the densities and $u_k(t)\ \forall k \in \{0, \dots, N-1\}$ the onramp flows resp. the ramp metering rates. Since we operate in quasi-steady-state, the sum off all inflows and outflows for every cell vanishes over one period:

$$\begin{aligned}
\rho_k(N) - \rho_k(0) &= \sum_{t=0}^{N-1}\left(\phi_{k-1}(t) + u_k(t) + \frac{\phi_k(t)}{\bar{\beta}_k}\right) \\
&= \sum_{i=0}^{N-1}\phi_{k-1}(t) + \sum_{i=0}^{N-1}u_k(t) + \sum_{i=0}^{N-1}\frac{\phi_k(t)}{\bar{\beta}_k} \\
&= N \cdot \left(\phi_{k-1}^{avg} + \bar{u}_k + \frac{\phi_k^{avg}(t)}{\bar{\beta}_k}\right) \stackrel{!}{=} 0
\end{aligned}$$

It follows that the average flows satisfy the steady-state conservation equation $\phi_{k-1}^{avg} + \bar{u}_k + \frac{\phi_k^{avg}(t)}{\bar{\beta}_k} = 0$. They do not necessarily satisfy the flow-density relationship given by the fundamental diagram, but they satisfy a related inequality instead:

$$\begin{aligned}\phi_k^{avg} &= \frac{1}{N}\sum_{t=0}^{N-1}\phi_k(t) = \frac{1}{N}\sum_{t=0}^{N-1}\min\left\{\bar\beta_k v_k\rho_k(t), F_k, (\bar\rho_{k+1}-\rho_{k+1}(t))w_{k+1}\right\}\\ &\le \min\left\{\frac{1}{N}\sum_{t=0}^{N-1}\bar\beta_k v_k\rho_k(t), F_k, \frac{1}{N}\sum_{t=0}^{N-1}(\bar\rho_{k+1}-\rho_{k+1}(t))w_{k+1}\right\}\\ &= \min\left\{\bar\beta_k v_k\rho_k^{avg}, F_k, (\bar\rho_{k+1}-\rho_{k+1}^{avg})w_{k+1}\right\}\end{aligned}$$

Similarly $\phi_0^{avg} \le \min\{(\bar\rho_1-\rho_1^{avg})w_1, F_0\}$ and $\phi_n^{avg} \le \min\left\{\bar\beta_n v_n\rho_n^{avg}, F_n\right\}$. It is now easy to show that there exists an equilibrium that achieves at least the same average throughput. Define:

$$\begin{aligned}u_k^{ss} &:= u_k^{avg},\\ \phi_0^{ss} &:= \phi_0^{avg}.\end{aligned}$$

Note that because the average quantities satisfy the steady-state conservation equation, $\phi_{k+1}^{ss} := \left(\phi_k^{ss}+u_{k+1}^{ss}\right)\bar\beta_{k+1}$ holds, i.e. the average flows define a valid traffic demand. Furthermore, $\phi_k^{ss} \le F_k$ holds. Therefore, the traffic demand is feasible. But we have previously shown in Lemmas 1 and 2, that for any feasible traffic demand u^{ss}, we can find densities to construct an equilibrium to the CTM. Note that the average onramp flows are obviously bounded by the minimal/ maximal onramp inflows during one cycle. This simple, yet important observation implies that the equilibrium onramp flows $u_k^{ss} = u_k^{avg}$ satisfy any box constraints that are satisfied by the onramp inflows during one cycle: $\underline{u}_k \le \min_{0\le t<N} u_k(t) \le u_k^{avg} \le \max_{0\le t<N} u_k(t) \le \bar u_k$. Therefore, the ramp metering strategy $u_k(t) = u_k^{avg}\ \forall t \ge 0$ is implementable for any cycle with average metering rates u_k^{avg}. Since the CTM is globally asymptotically stable (and the equilibrium flows are unique), convergence to an equilibrium with equilibrium flows equalling the average flows over one period of the cycle, i.e. $\lim_{t\to\infty}\phi_k(t) = \phi_k^{avg}$, is guaranteed. □

Having thus established that there does not exist an improving cycle, the problem of maximizing long–term throughput under constant boundary conditions is reduced to optimizing over the set of equilibria.

2.2.3 Flow-Optimal Equilibria

In this section, we will address the problem of optimizing over steady-state equilibria of the CTM. To this end, we will first state the (nonconvex) main problem, show the equivalence of this problem with a suitable linear relaxation and then use duality arguments to derive optimality conditions.

Consider a highway modeled by the CTM and controlled by ramp metering. We want to find the equilibrium that maximizes a positive combination of all flows $c^\top\phi$ ($c \in \mathbb{R}_+^{n+1}$) by choosing appropriate (steady-state) ramp metering rates u_k. The ramp metering rates are assumed to be constrained by box constraints $\underline{u}_k \le u_k \le \bar u_k$. In the simplest case, the lower limit might be equal to zero to prevent negative flows and the upper bounds will reflect the maximal number of cars that want to enter the freeway at a certain onramp per time period. We assume that the lower bounds $(u_0, \underline{u}_1, \ldots, \underline{u}_n)$ on the inflows are onramp-feasible, guaranteeing the existence of a feasible solution. Thus we consider the following optimization problem over equilibria of the CTM:

$$\begin{aligned}\underset{u,\phi,\rho}{\text{maximize}}\quad & c^\top\phi\\ \text{subject to}\quad & \underline{u}_k \le u_k \le \bar u_k,\quad 1\le k\le n,\\ & (\phi,\rho)\in\mathcal{E}(u).\end{aligned}\tag{2.4}$$

Note that Problem (2.4) includes the maximization of the Total Travel Distance TTD $:= \sum_{k=0}^{n} \frac{\phi_k}{\beta_k}$ or the total discharge flows $r_{tot} := \phi_n + \sum_{k=1}^{n} \frac{\beta_k}{\bar{\beta}_k}\phi_k$ as special cases. Also note that Problem (2.4) is nonconvex, due to the nonlinear flow constraints that make the set of equilibria $\mathcal{E}(u)$ a nonconvex set.

Even though the flow-constraints are nonconvex, it is well-known that there exists an uncongested, flow-maximizing equilibrium for freeways under fairly general conditions Wattleworth (1965); Chen et al. (1974). One can find this solution by solving a suitable Linear Program, for example by introducing a condition that restricts the highway to free-flow conditions (which does not introduce any conservativeness in terms of the objective value, as long as the problem remains feasible). We want to avoid this explicit restriction and consider the following relaxation:

$$\begin{aligned}
\underset{u,\phi}{\text{maximize}} \quad & c^\top \phi \\
\text{subject to} \quad & \underline{u}_k \le u_k \le \bar{u}_k, && 1 \le k \le n, \\
& \phi_k = (\phi_{k-1} + u_k) \cdot \bar{\beta}_k, && 1 \le k \le n, \\
& \phi_k \le F_k, && 1 \le k \le n, \\
& \phi_0 \le \tilde{F}_0 := \min\{F_0, u_0\},
\end{aligned} \tag{2.5}$$

in which the nonconvex flow-constraints have been relaxed and the constraints involving the densities have been removed altogether.

Proposition 1. *Let $c \in \mathbb{R}_+^{n+1}$. Then problem (2.4) is equivalent to the relaxation (2.5), in the sense that the objective values are equal. Furthermore, given a maximizer (u^*, ϕ^*) of the relaxed problem (2.5), we can compute densities ρ^* such that (u^*, ϕ^*, ρ^*) is a maximizer of (2.4).*

Proof. Since (2.5) is a relaxation of (2.4), it is sufficient to show that for any optimizer ϕ^*, u^* to (2.5), we can find ρ^* such that ϕ^*, u^*, ρ^* are feasible for (2.4). Assume u^*, ϕ^* are solutions of the relaxed problem (2.5). Then ϕ^* is also the solution of $\phi^* = \underset{\phi}{\text{argmax}}\{c^\top \phi : \phi \in \mathcal{F}\}$ with:

$$\mathcal{F} := \left\{ \begin{array}{ll} \phi_k = (\phi_{k-1} + u_k^*) \cdot \bar{\beta}_k & 1 \le k \le n \\ \phi_k \le F_k & 0 \le k \le n \end{array} \right. .$$

For fixed onramp flows, all flows ϕ_k can be expressed as affine functions of ϕ_0, thus we can rewrite:

$$\begin{aligned}
\phi^* &= \underset{\phi}{\text{argmax}}\{c^\top \phi : \phi \in \mathcal{F}\} && = \underset{\phi}{\text{argmax}}\{a\phi_0 + b : \phi \in \mathcal{F}\} \\
&= \underset{\phi}{\text{argmax}}\{\phi_0 : \phi \in \mathcal{F}\} && = \phi(u^*)
\end{aligned}$$

for suitable $a, b \in \mathbb{R}_+$. We see that the flows ϕ^* equal the equilibrium flows for the fixed traffic demands u^* as stated in Remark 1. We have previously established that the set of equilibrium densities is nonempty in every case, according to Lemmas 1 and 2. Therefore, we can always find feasible equilibrium densities ρ^* and equivalence of the optimization problems holds. □

Note that in general, the optimal solution set for the densities includes equilibria in which some cells are congested. While optimizing over free-flow conditions will yield a global optimum, the CTM also allows for (partly) congested equilibria, which achieve the same objective value.

We will now derive sufficient optimality conditions for the maximal flow problem (2.4). These optimality conditions will be tailored for the analysis of distributed control approaches and are based on the following consideration: Assume a distributed controller with the objective of moving the local density at every controlled onramp (as close as possible) to the critical density. It is easy to check that each individual flow will be maximal if $\rho_k = \rho_k^c$ is achieved in every cell, which is the main idea behind this control approach. However, this will in general not be the case due to saturation of the available control inputs (the ramp metering rates), in particular if multiple local controllers interact while controlling a freeway. Therefore, the performance of such closed-loop equilibria in terms of flow maximization is not clear a priori. The following theorem presents sufficient optimality conditions, by imposing only local constraints on the ramp metering rates, dependent only on the local densities:

Theorem 2. *Let u, ϕ, ρ be an equilibrium of the CTM and assume the equilibrium ramp metering u rates satisfy:*

$$\begin{aligned} &\text{“free-flow”:} && u_k = \bar{u}_k && \forall k \,:\, \rho_k < \rho_k^c, \\ &\text{“critical density”:} && \underline{u}_k \le u_k \le \bar{u}_k && \forall k \,:\, \rho_k = \rho_k^c, \\ &\text{“congestion”:} && u_k = \underline{u}_k && \forall k \,:\, \rho_k > \rho_k^c. \end{aligned} \tag{2.6}$$

Then u, ϕ, ρ solve the maximal flow problem (2.4).

Proof. Feasibility of the primal problem (2.4) holds by assumption. To verify optimality, it is sufficient to show that the equilibrium flows solve the LP-relaxation (2.5), according to Proposition 1. Consider the dual of problem (2.5) which is given as:

$$\begin{aligned} \underset{\mu,\nu,\xi,\lambda}{\text{minimize}} \quad & \nu^\top \underline{u} - \xi^\top \bar{u} - \lambda^\top F \\ \text{subject to} \quad & \mu_k + \nu_k - \xi_k = 0 && 1 \le k \le n \\ & -c_k + \tfrac{\mu_k}{\beta_k} - \mu_{k+1} + \lambda_k = 0 && 0 \le k \le n \\ & \mu_0 = \mu_{n+1} = 0 \\ & \mu \in \mathbb{R}^{n+2}, \nu \in \mathbb{R}_+^n, \xi \in \mathbb{R}_+^n, && \lambda \in \mathbb{R}_+^{n+1} \end{aligned} \tag{2.7}$$

To simplify notation, we have introduced $\mu_0 := 0$ and $\mu_{n+1} := 0$. We will show that for any solution to the primal problem which satisfies (2.6), we can construct a complementary dual solution, thus proving that the primal solution is indeed optimal. The complementarity conditions are given as

$$\begin{aligned} 0 \le (\underline{u} - u) \perp \nu \ge 0, \\ 0 \le (u - \bar{u}) \perp \xi \ge 0, \\ 0 \le (\phi - F) \perp \lambda \ge 0. \end{aligned}$$

For ease of notation, we define the following index sets: The set of all bottlenecks $B := \{k : F_k = \phi_k\}$, the set of all congested cells $C := \{k : \rho_k > \rho_k^c\}$, the set of all cells in free flow $F := \{k : \rho_k < \rho_k^c\}$ and the set of all cells in which the critical density is achieved $D := \{k : \rho_k = \rho_k^c\}$. By combining the constraints for dual feasibility and the complementarity constraints, we end up with the following optimality conditions:

$$\phi, \rho \in \mathcal{E}(u), \quad u, \rho \text{ satisfy (2.6)}, \tag{2.8a}$$

$$-c_k + \frac{\mu_k}{\bar{\beta}_k} - \mu_{k+1} + \lambda_k = 0, \tag{2.8b}$$

$$\mu_k = 0 \text{ if } k \in D, \quad \lambda_k = 0 \text{ if } k \notin B, \tag{2.8c}$$

$$\mu_k \ge 0 \text{ if } k \in F, \quad \mu_k \le 0 \text{ if } k \in C, \tag{2.8d}$$

$$\lambda_k \ge 0 \text{ if } k \in B. \tag{2.8e}$$

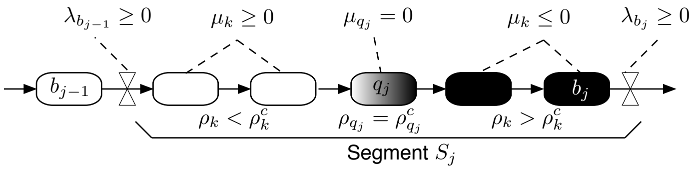


Figure 2.5: Dual variables corresponding to the congestion/ free-flow pattern of the freeway within one segment.

We will now find an explicit solution to the dual problem, by constructing a solution to every segment S_j. Define

$$\mu_{q_j} := 0.$$

Then feasible multipliers μ_k for all other cells in segment S_j can be computed by iterating in upstream/downstream direction, starting from the cell $b_j + n_F$:

$$\begin{aligned} \mu_k &:= \bar{\beta}_k \left(\mu_{k+1} + c_k\right), & b_{j-1} < k < q_j, \\ \mu_k &:= \frac{\mu_{k-1}}{\bar{\beta}_{k-1}} - c_{k-1}, & q_j < k \leq b_j, \end{aligned}$$

and the multipliers λ are computed as

$$\begin{aligned} \lambda_k &:= c_k - \frac{\mu_k}{\bar{\beta}_k} + \mu_{k+1}, & k \in B, \\ \lambda_k &:= 0, & k \notin B. \end{aligned}$$

The resulting pattern within one segment is depicted in Figure 2.5.

We will now verify that the proposed solution indeed solves the optimality conditions (2.8). Note that condition (2.8a) is satisfied by assumption. Conditions (2.8b) and (2.8c) are satisfied by construction. Conditions (2.8d) and (2.8e) remain to be checked.

We will first show that conditions (2.8d) hold for a generic segment S_j. For this we need some intermediate steps:

- Recall that the parameters $\bar{\beta}_k$ and c_k are nonnegative. We claim that the multipliers of all cells upstream of cell q_j, but within segment S_j, are nonnegative. This can be verified by induction: $\mu_{q_j} = 0$ by definition. Assume now that $\mu_k \geq 0$. Then $\mu_{k-1} = \bar{\beta}_{k-1}\left(\mu_k + c_{k-1}\right) \geq 0$ for all cells with indices $b_{j-1} < k < q_j$.

- Conversely, the multipliers of all cells downstream of cell q_j, but within segment S_j, are nonpositive. This can again be verified by induction: $\mu_{q_j} = 0$ by definition. Assume now that $\mu_k \leq 0$. Then $\mu_{k+1} = \frac{\mu_k}{\bar{\beta}_k} - c_k \geq 0$ for all cells with indices $q_j < k \leq b_j$.

Recall now that according to Lemma 3, the congestion pattern within a segment is ordered. In particular, all cells $b_{j-1} < k < q_j$ operate in free-flow and all cells $q_j < k \leq b_j$ are congested. Satisfaction of conditions (2.8d) immediately follows.

We can also conclude from the previous analysis that for every bottleneck b_j, it holds that $\mu_{b_j} \leq 0$ (since cell b_j is located downstream of cell q_j) and also $\mu_{b_j+1} \geq 0$ (since cell $b_j + 1$ is located upstream of cell $q_j + 1$). Therefore $\lambda_{b_j} := c_{b_j} - \frac{\mu_{b_j}}{\bar{\beta}_{b_j}} + \mu_{b_j+1} \geq 0$ and conditions (2.8e) follow. □

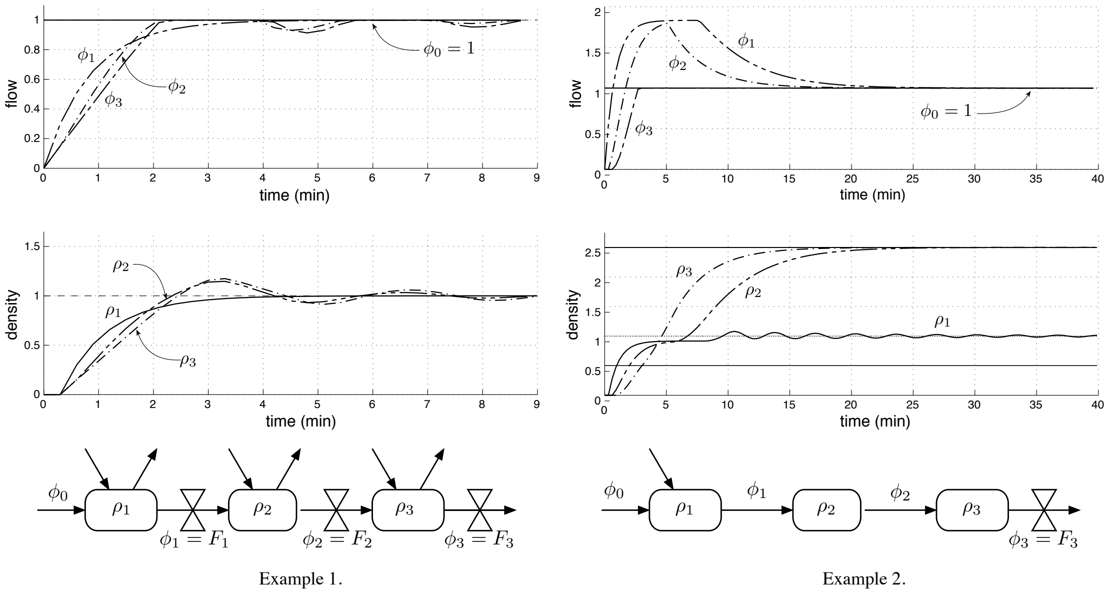


Figure 2.6: Simulation results using the CTM. Note that the flows are normalized such that a flow of "1" means that the optimal steady-state flow (computed separately) is achieved. Similarly, the densities are normalized by the respective critical densities. Note the different timescales.

It is interesting to note that the result does not depend on the choice of the weights for the individual mainline flows c (as long as all are non-negative), so all such objectives will be jointly maximized by an equilibrium satisfying the optimality condition (2.6). This is a consequence of the assumption of constant split rations β_k, under which it is impossible to trade off the mainline flow at some part of the freeway with the mainline flow at another place.

2.3 Application

To demonstrate the practical relevance of the previously derived optimality conditions, we use them to analyze the well-known distributed ramp-metering strategy ALINEA Papageorgiou et al. (1991). ALINEA is in essence an integral controller that aims at stabilizing the local densities at the critical density. It is easy to verify that every closed-loop equilibrium of a freeway controlled by local ALINEA controllers satisfies the optimality conditions (2.6).

We implement the standard ALINEA anti-windup integral controller

$$u_k(t) := \tilde{u}_k(t-1) + K_I \cdot (\rho_k^c - \rho_k(t))$$

in which $\tilde{u}_k(t-1)$ is the saturated inflow $\tilde{u}_k(t) := \max(\underline{u}_k, \min(u_k(t), \bar{u}_k))$ and $K_I := 70/\rho_k^c$ is the integral gain chosen as recommended in Papageorgiou et al. (1991). Closed-loop simulation results of this controller operating on simple freeways represented by the CTM are depicted in Figure 2.6. In both examples, the closed-loop system converges to the optimal closed-loop equilibrium, as predicted. In the first example, the controller converges to the unique uncongested equilibrium. In fact, all densities converge to the respective critical densities. In the second example, however, the controlled onramp and the bottleneck are two sections apart. We observe convergence to a partly congested equilibrium, also the convergence is slow and heavy oscillations occur. These effects have already been described Wang

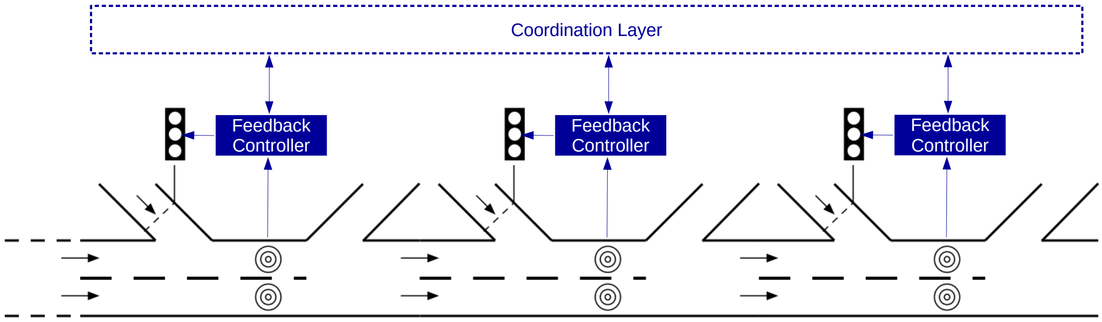


Figure 2.7: Conceptual architecture of coordinated ramp metering.

et al. (2014), nevertheless, the optimal flows are achieved in the limit as long as we use the CTM for simulations.

It is important to keep in mind that this result holds only under the assumptions made in the CTM. In particular, the equilibrium may contain congested bottlenecks, which lead in practice to an empirically observed reduction of the bottleneck capacity. Such effects are not described by the simplistic CTM. In this sense, the simulations described in this section are not meant to be a realistic assessment of the real-world traffic behavior, but rather a way to demonstrate the properties of the idealized CTM. Note however, that the equilibrium densities are not unique and techniques described in Gomes et al. (2008) or Wang et al. (2014) can be employed to steer the system to the preferred equilibrium (usually the unique, least congested equilibrium) in order to avoid the aforementioned capacity drop.

2.4 Integration within SPEEDD

Its inherently decentralized structure makes the ALINEA controller perfectly suitable for a distributed architecture. In addition, it does not rely on online optimization at all. As a (nonlinear) feedback controller, decisions at runtime are made by evaluating a function, which has a constant complexity ($\mathcal{O}(1)$), irrespective of the size of the road network. Therefore, it has been selected for a first implementation within the SPEEDD project. It is intended to serve as a bottom-level controller for the freeway use case, on top of which coordinated, optimization–based decision making strategies can be implemented (Figure 2.7).

We also want to briefly address the question about the appropriate degree of automation. In principle, there are two modes of operation envisioned for SPEEDD, as outlined for example in D8.1:

- **Decision support** Decision making only *suggests* appropriate control actions. At the very least, every proposed action has to be reviewed and acknowledged by a (human) operator.
- **Automatic decision making** Decisions are implemented automatically, that is they are immediately sent to the actuators. The operators will receive a notification, though, and might have the possibility to override the automatically chosen actions.

A low-level ramp metering controller such as ALINEA will change ramp metering rates typically every minute. This will happen for every onramp independently, also one can expect most changes of the ramp metering rates to be relatively small, at least in the absence of disruptive events. A manual acknowledgement of all those changes seems to be neither practical nor desirable. In the first implementation,

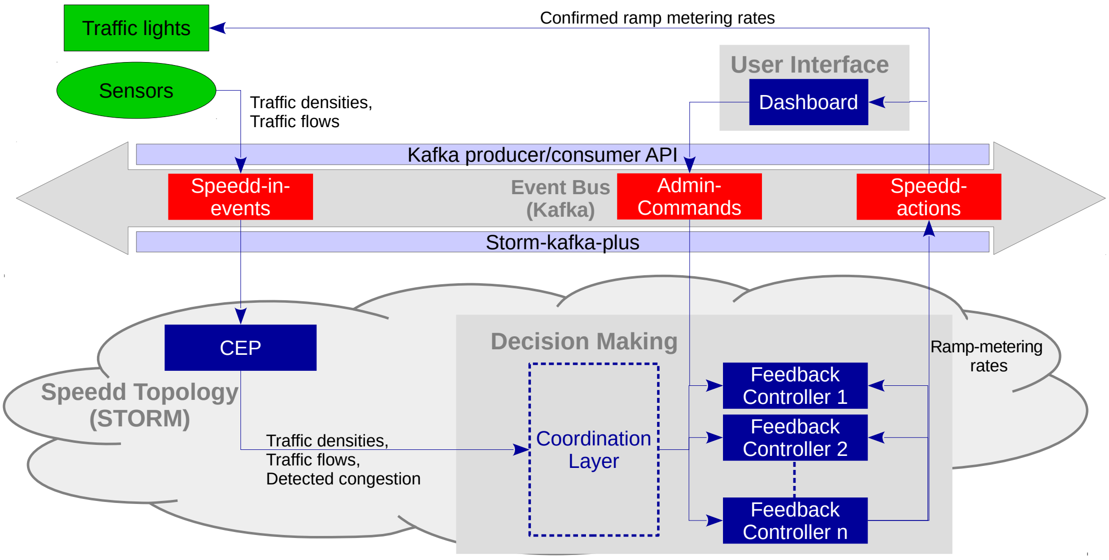


Figure 2.8: The integration of the described decision making unit for freeway traffic. At the current stage, local feedback controllers and the communication with the user interface are in place. The development and implementation of a coordination layer will be the next step.

we considered those changes to be automatic decisions that are sent to the actuators directly, without requiring prior approval. However, for a system like a traffic network, an option for human operators to interfere is mandatory, at the very least as a response to emergencies. The current implementation, as depicted in Figure 2.8, allows for this by providing the possibility to set lower and upper limits on the metering rates, for every individual onramp (”Admin Commands”). Those limits, which correspond to $\underline{u}_k$ and $\bar{u}_k$ in the previous analysis, will be respected by all automatically made decisions, until they are manually changed or removed by the operators. It shall be stressed, that the previously derived optimality conditions continue to hold in the presence of (constant) limits on the ramp metering rates. Setting the lower and upper limits to equal values, $\underline{u}_k = \bar{u}_k$, effectively disables the automatic controller at the respective onramp. Thus the operators have the option to override the automatic control entirely at any time, if the situation seems to require such a drastic action.

2.5 Conclusions

In this chapter, we have conducted a theoretical analysis of the CTM for freeways. We found simple optimality conditions for flow–maximization, that are tailored to distributed control approaches, since only constraints between local ramp metering rates and the densities in the adjacent cells are imposed. In particular, the optimality conditions are satisfied for all closed-loop equilibria of a freeway controlled by independent local ALINEA controllers, thus proving that all such equilibria are globally optimal (w.r.t. flow maximization). Due to its simplicity, the ALINEA controller has been chosen for a first implementation in the SPEEDD use-case. Based on the presented analysis, the next step will be the derivation and implementation of a hierarchical control scheme, that uses (robust) online optimization in order to coordinate the lower level feedback controllers.

3

An efficient one-step-ahead optimal control for urban signalized traffic networks based on an averaged Cell-Transmission model

P. Grandinetti, F. Garin, C. Canudas de Wit

Our contribution to the Real-time event-based decision-making under uncertainty work package consists in the design of a novel algorithm for optimization of large–scale urban traffic network Grandinetti et al..

The steadily increasing traffic demands have given rise to the need for efficient network operations. In this sense, traffic lights assume a fundamental role, since they are the major control measure in urban scenario.

Urban traffic control strategies are classified as *fixed–time* techniques and *model–based* algorithms. The main drawback of the former ones is that their settings are based on historical rather than real–time data, while the latter ones basic problem is the presence of discrete variables that require exponential complexity algorithms for a global optimization.

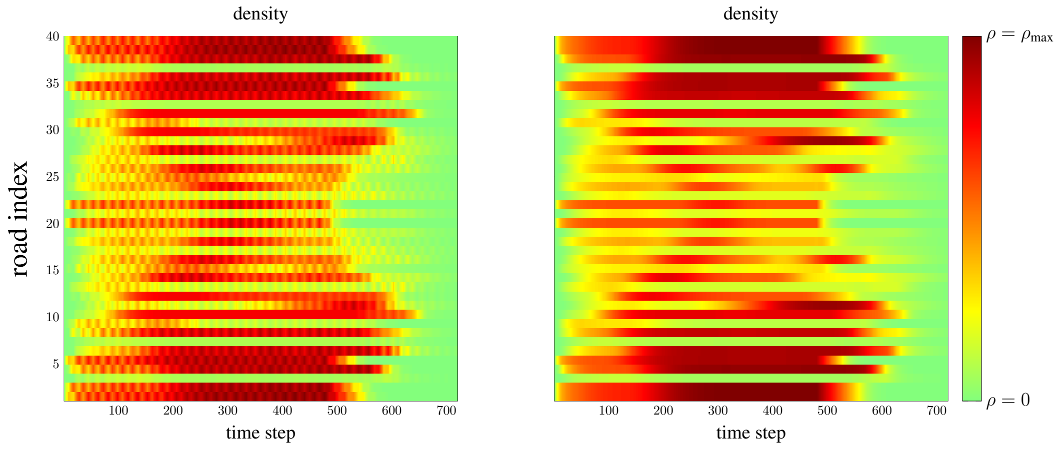
We have instead developed a decision–making scheme that can be implemented via linear optimization, and it is therefore computationally very efficient and scalable. This technique is based on a dynamical representation of traffic flow inside the newwork.

3.1 Traffic flow model

Traffic evolution in time and space is a complex system which strongly affects security and pollution; hence, effective and easy–to–handle models are needed to represent and control its behaviour.

The scientific community relies on macroscopic models of time–space evolution of the traffic. Such models describe traffic as a fluid, and are based on a mass conservation law . With respect to microscopic models, macroscopic ones are preferred due to their simplicity and accuracy in characterizing vehicles' flows and densities. The Cell Transmission Model (CTM) is an example, widely used, of this kind of representation.

We have build a model for large signalized traffic network based on the CTM, where flows at intersection of roads are regulated by traffic lights, and we have introduced the concept of averaged CTM, a



(a) (b)

Figure 3.1: Precision of the averaged CTM approved via software simulation. In 3.1a (regarding the actual system) and in 3.1b (regarding the averaged system) each row shows the density of a road evolving in time.

more effective characterization of the system by means of control purpose. The averaged CTM evolution, tested in software simulation within a network with 40 roads, results to also have a good precision in terms of reliability with respect to the actual CTM network, as figure 3.1 shows.

3.2 Traffic performance

Traffic behaviour needs to be evaluated and assessed with respect to performance indices properly defined. There exist several metrics in literature to address performance evaluation; we focused on the following two features:

Service of Demand An urban traffic network is an highly dynamical environment that continously receives demand from outside. This demand cannot be ignored just to favour the inner quality of the system, because the external request will end up growing with several undesired effects, due to the bigger and bigger queues arising outside.

For this reason we define as service of demand the number of vehicles (users) served:

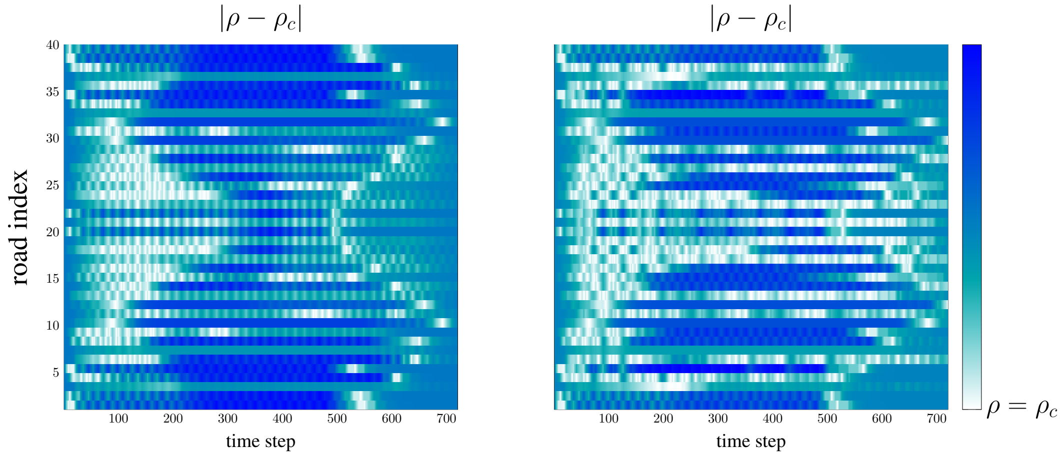
$$\mathrm{SoD}(t) = \int_0^t \varphi_r^{\mathrm{in}}(\tau)d\tau$$

where φ_r^{in} is the flow at the network boundary which enters in road r. The Service of Demand is a quantity that we would like to maximize.

Optimization of the infrastructures In urban networks some roads are preferred than others by the users. The infrastructure holder would like to set traffic lights as to diminish this usage disparity, to guarantee a more equilibrate diffusion of vehicles, thus reducing hard congestions in main streets as well as the possibility of accidents.

A standard metric that takes into account this behaviour is the Total Travel Distance, a cumulative index here defined as:

$$\mathrm{TTD}(t) = \int_0^t \Big(\sum_{r \in \mathrm{network}} \varphi_r(\tau) \Big) d\tau$$



(a) (b)

Figure 3.2: Application of the proposed decision–making strategy. Figures 3.2a and 3.2b show the distance from the best working point (called *critical density*, ρ_c) for the system with fixed strategy and with our algorithm, respectively. Lighter color means better performance.

Table 3.1: Improvement in the network with the proposed control strategy.

Index	Improvement (%)
SoD	5.5 (per entering road)
TTD	14.6

where φ_r is the flow inside the road r.

3.3 Decision–making strategy

We developed a strategy for deciding the duty cycle to be assigned to the traffic lights using a one–step–ahead control. The optimization problem we stated can be solved by means of linear programming, and it is therefore very suitable for practical purpose. The size of the Linear Program scales linearly ($\mathcal{O}(n)$) with the number of cells in the CTM of the road network. Thanks to this efficient optimization we can set duty cycles periodically, achieving good performance improvements for the two metrics previously defined. The effectiveness of our algorithm was tested in software simulation and compared to a fixed–decision strategy.

Representative results are showed in Figure 3.2 and in Table 3.1. Note that:

- Our algorithm achieves good performance regarding the optimization of the infrastructures. Figure 3.2 shows how far each road's density is from its best working point ρ_c, where lighter color means better performance;

- Table 3.1 gives the quantitative measures of the improvement, which is positive for both the choosen indices.

4

Distributionally Robust Classifiers

The problem of credit card fraud detection is a heavily unbalanced classification problem, since only a small minority of transactions are fraudulent. It is well known that unbalanced data pose a challenge for machine learning. Common state of the art solutions include over-/ under-sampling as well as the replacement of individual data points by probability distributions, which approximate the data prior to training. This approach has recently been extended by exploiting results from robust optimization to create distributionally robust classifiers (Van Parys et al. (2014); Stellato (2014)). A brief summary of the derivation of distributionally robust classifiers will be provided in the following.

4.1 Problem Formulation

Robust optimization addresses optimization problems, in which at least one of the problem parameters is a priori uncertain and will only be revealed after a decision has been made. In robust optimization, it is assumed that even though we do not know the value of those parameters a priori, we know that they belong to a given set and we seek to optimize the worst case outcome.

Consider the problem to separate samples drawn from two probability distributions by a hyperplane, such that the probability of misclassification is minimized for either side. Assume further that the mean and the covariance of the probability distributions are known, but not the complete probability density function. The problem of minimizing the classification error can then be formulated as a robust optimization problem, in which we find a hyperplane such that we minimize the classification error for the worst-case probability distribution that coincides with the given first and second moments. In practice, it often seems to be unjustified to allow for entirely arbitrary probability density functions. In particular, many probability distributions encountered in practice are unimodal. Intuitively, unimodality of the probability density function means that obtaining samples ”close” to the mean is more likely than obtaining samples ”further away” from the mean.

Let the μ_1 and μ_2 denote the mean of the first and the second probability distribution, and likewise let S_1 and S_2 denote the respective variances. By $\mathbb{P} \in \mathcal{P}(\mu, S) \cap \mathcal{P}_\alpha$ we denote a probability distribution $\mathbb{P}$ that has mean μ and variance S, and an α-unimodal probability density function[1]. The problem of

[1] In Van Parys et al. (2014), α-unimodality is introduced as a generalization of the concept of unimodality.

minimizing the worst-case classification error can be formulated as:

$$\begin{aligned} \max_{a,\gamma,b} \quad & \gamma \\ \text{subject to} \quad & \mathbb{P}(-a^\top x > -b) \leq 1-\gamma, \quad \forall \mathbb{P} \in (\mu_1, S_1) \cap \mathcal{P}_\alpha \\ & \mathbb{P}(+a^\top x > +b) \leq 1-\gamma, \quad \forall \mathbb{P} \in (\mu_2, S_2) \cap \mathcal{P}_\alpha \end{aligned} \tag{4.1}$$

4.2 Results

The previously introduced optimization problem includes an infinite number of constraints, because of the ”for all” classifier, therefore a straightforward numerical solution is impossible. However, by employing results from Van Parys et al. (2014), Stellato (2014) reformulates the problem as an equivalent, nonlinear, but finite dimensional problem:

$$\begin{aligned} \min_{a,b} \quad & \omega \\ \text{subject to} \quad & (-b + a^\top \mu_1) \geq ||S_1^{\frac{1}{2}} a|| \\ & (-b + a^\top \mu_2) \geq ||S_2^{\frac{1}{2}} a|| \end{aligned}$$

This problem can be reformulated into a finite dimensional Second-order Cone Problem (SOCP):

$$\begin{aligned} \min_{a} \quad & ||S_1^{\frac{1}{2}} a|| + ||S_2^{\frac{1}{2}} a|| \\ \text{subject to} \quad & a^\top (\mu_1 - \mu_2) = 1 \end{aligned} \tag{4.2}$$

Note in particular that this problem is convex. Numerical solvers for SOCPs exist, so that the resulting optimization problem can be solved (numerically) for the optimizer a^*. The remaining quantities can be computed as

$$\omega^* = \frac{||S_1^{\frac{1}{2}} a^*|| + ||S_2^{\frac{1}{2}} a^*||}{a^\top (\mu_1 - \mu_2)}$$

and

$$b^* = a^* \mu_1 - \omega^* ||S_1^{\frac{1}{2}} a^*||$$

Note that the objective has been the minimization of the classification error for either probability distribution. A tractable problem formulation for the objective of minimizing the overall misclassification error is also given in Stellato (2014).

4.3 Future work

Distributionally robust classifiers have successfully been applied to relatively small benchmark problems. In order to facilitate application to big–data applications like Credit Card Fraud Detection, it might be necessary to revise the existing codebase. An immediate follow–up question is whether the distributionally robust classifiers can be generalized for nonlinear decision boundaries. In machine learning, this is commonly achieved by the “kernel trick”. However, the efficient application of the standard kernel trick requires a special structure of the machine learning problem, which is not shared with the prototype problem 4.1. It seems unlikely that the standard Kernel trick will be applicable, but alternatives that approximate the benefits of standard kernel methods have recently been proposed. We intend to address the question of whether the proposed classifiers can be extended to a kernel method in the near future.

5

Conclusions

In this document, we have described an initial body of decision making tools for freeway traffic control, inner-city traffic control and credit card fraud detection:

For freeway traffic, we find simple optimality conditions that hold for all closed-loop equilibria of a freeway controlled by independent local ALINEA controllers, thus proving that all such equilibria are globally optimal (w.r.t. flow maximization). It is well-known that in practice, there exist effects that are not modeled by the CTM, which require coordination and online-optimization of the decisions. Such a coordinated, optimization based decision making algorithm that builds upon the low level feedback control is expected to be developed during the next steps of the SPEEDD project.

For inner city traffic, we developed a strategy for deciding the duty cycle to be assigned to the traffic lights using a one–step–ahead control. The optimization problem we stated can be solved by means of linear programming, and it is therefore very suitable for practical purpose.

Distributionally robust classifiers (in their present form) are linear classifiers. In a next step, we intend to assess the performance of distributionally robust classifiers on the SPEEDD use-case of Credit Card Fraud Detection, in comparison to other classifiers. To achieve competetiveness, it might also be necessary to explore if distributionally robust classifiers, which are linear classifiers in their present form, can be extended to nonlinear classifiers by employing the kernel trick or an equivalent extension.

The development of decision making components is ongoing work, and future deliverables (D4.2 and D4.3) will build upon the initial results presented in this document.

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